6.2 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. In a right triangle, with legs a and b and hypotenuse c, the Pythagorean Theorem states that ________. (p. 30)

2. The value of the function $f(x) = 3x - 7$ at 5 is ________. (pp. 218–226)

Concepts and Vocabulary

3. Two acute angles whose sum is a right angle are called ________.

4. The sine and ________ functions are cofunctions.

5. $\tan 28° = \cot$ ________.

6. For any angle θ, $\sin^2\theta + \cos^2\theta =$ ________.

7. *True or False:* $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

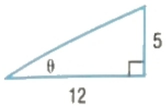
8. *True or False:* $1 + \tan^2\theta = \csc^2\theta$.

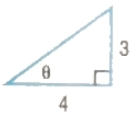
9. *True or False:* If θ is an acute angle and $\sec\theta = 3$, then $\cos\theta = \dfrac{1}{3}$.

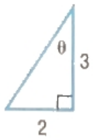
10. *True or False:* $\tan\dfrac{\pi}{5} = \cot\dfrac{4\pi}{5}$.

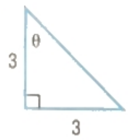
Exercises

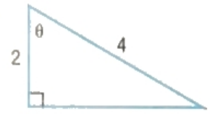
In Problems 11–20, find the value of the six trigonometric functions of the angle θ in each figure.

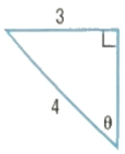
11. 

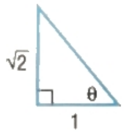
12. 

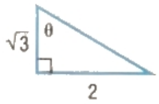
13. 

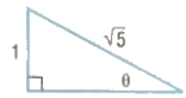
14. 

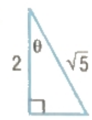
15. 

16. 

17. 

18. 

19. 

20. 

In Problems 21–24, use identities to find the exact value of each of the four remaining trigonometric functions of the acute angle θ.

21. $\sin\theta = \dfrac{1}{2}$, $\cos\theta = \dfrac{\sqrt{3}}{2}$

22. $\sin\theta = \dfrac{\sqrt{3}}{2}$, $\cos\theta = \dfrac{1}{2}$

23. $\sin\theta = \dfrac{2}{3}$, $\cos\theta = \dfrac{\sqrt{5}}{3}$

24. $\sin\theta = \dfrac{1}{3}$, $\cos\theta = \dfrac{2\sqrt{2}}{3}$

In Problems 25–36, use the definition or identities to find the exact value of each of the remaining five trigonometric functions of the acute angle θ.

25. $\sin\theta = \dfrac{\sqrt{2}}{2}$

26. $\cos\theta = \dfrac{\sqrt{2}}{2}$

27. $\cos\theta = \dfrac{1}{3}$

28. $\sin\theta = \dfrac{\sqrt{3}}{4}$

29. $\tan\theta = \dfrac{1}{2}$

30. $\cot\theta = \dfrac{1}{2}$

31. $\sec\theta = 3$

32. $\csc\theta = 5$

33. $\tan\theta = \sqrt{2}$

34. $\sec\theta = \dfrac{5}{3}$

35. $\csc\theta = 2$

36. $\cot\theta = 2$

In Problems 37–54, use Fundamental Identities and/or the Complementary Angle Theorem to find the exact value of each expression. Do not use a calculator.

37. $\sin^2 20° + \cos^2 20°$

38. $\sec^2 28° - \tan^2 28°$

39. $\sin 80° \csc 80°$

40. $\tan 10° \cot 10°$

41. $\tan 50° - \dfrac{\sin 50°}{\cos 50°}$

42. $\cot 25° - \dfrac{\cos 25°}{\sin 25°}$

43. $\sin 38° - \cos 52°$

44. $\tan 12° - \cot 78°$

45. $\dfrac{\cos 10^\circ}{\sin 80^\circ}$

46. $\dfrac{\cos 40^\circ}{\sin 50^\circ}$

47. $1 - \cos^2 20^\circ - \cos^2 70^\circ$

48. $1 + \tan^2 5^\circ - \csc^2 85^\circ$

49. $\tan 20^\circ - \dfrac{\cos 70^\circ}{\cos 20^\circ}$

50. $\cot 40^\circ - \dfrac{\sin 50^\circ}{\sin 40^\circ}$

51. $\tan 35^\circ \cdot \sec 55^\circ \cdot \cos 35^\circ$

52. $\cot 25^\circ \cdot \csc 65^\circ \cdot \sin 25^\circ$

53. $\cos 35^\circ \sin 55^\circ + \cos 55^\circ \sin 35^\circ$

54. $\sec 35^\circ \csc 55^\circ - \tan 35^\circ \cot 55^\circ$

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55. Given $\sin 30^\circ = \dfrac{1}{2}$, use trigonometric identities to find the exact value of
(a) $\cos 60^\circ$
(b) $\cos^2 30^\circ$
(c) $\csc \dfrac{\pi}{6}$
(d) $\sec \dfrac{\pi}{3}$

56. Given $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$, use trigonometric identities to find the exact value of
(a) $\cos 30^\circ$
(b) $\cos^2 60^\circ$
(c) $\sec \dfrac{\pi}{6}$
(d) $\csc \dfrac{\pi}{3}$

57. Given $\tan \theta = 4$, use trigonometric identities to find the exact value of
(a) $\sec^2 \theta$
(b) $\cot \theta$
(c) $\cot\left(\dfrac{\pi}{2} - \theta\right)$
(d) $\csc^2 \theta$

58. Given $\sec \theta = 3$, use trigonometric identities to find the exact value of
(a) $\cos \theta$
(b) $\tan^2 \theta$
(c) $\csc(90^\circ - \theta)$
(d) $\sin^2 \theta$

59. Given $\csc \theta = 4$, use trigonometric identities to find the exact value of
(a) $\sin \theta$
(b) $\cot^2 \theta$
(c) $\sec(90^\circ - \theta)$
(d) $\sec^2 \theta$

60. Given $\cot \theta = 2$, use trigonometric identities to find the exact value of
(a) $\tan \theta$
(b) $\csc^2 \theta$
(c) $\tan\left(\dfrac{\pi}{2} - \theta\right)$
(d) $\sec^2 \theta$

61. Given the approximation $\sin 38^\circ \approx 0.62$, use trigonometric identities to find the approximate value of
(a) $\cos 38^\circ$
(b) $\tan 38^\circ$
(c) $\cot 38^\circ$
(d) $\sec 38^\circ$
(e) $\csc 38^\circ$
(f) $\sin 52^\circ$
(g) $\cos 52^\circ$
(h) $\tan 52^\circ$

62. Given the approximation $\cos 21^\circ \approx 0.93$, use trigonometric identities to find the approximate value of
(a) $\sin 21^\circ$
(b) $\tan 21^\circ$
(c) $\cot 21^\circ$
(d) $\sec 21^\circ$
(e) $\csc 21^\circ$
(f) $\sin 69^\circ$
(g) $\cos 69^\circ$
(h) $\tan 69^\circ$

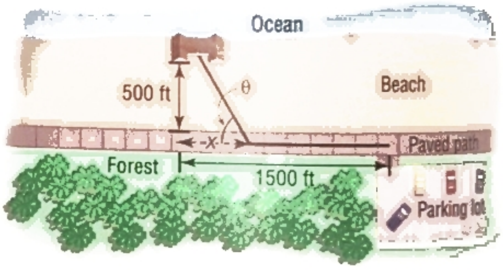
63. If $\sin \theta = 0.3$, find the exact value of $\sin \theta + \cos\left(\dfrac{\pi}{2} - \theta\right)$.

64. If $\tan \theta = 4$, find the exact value of $\tan \theta + \tan\left(\dfrac{\pi}{2} - \theta\right)$.

65. Find an acute angle θ that satisfies the equation $\sin \theta = \cos(2\theta + 30^\circ)$.

66. Find an acute angle θ that satisfies the equation $\tan \theta = \cot(\theta + 45^\circ)$.

67. **Calculating the Time of a Trip** From a parking lot you want to walk to a house on the ocean. The house is located 1500 feet down a paved path that parallels the beach, which is 500 feet wide. Along the path you can walk 300 feet per minute, but in the sand on the beach you can only walk 100 feet per minute. See the illustration.



(a) Calculate the time T if you walk 1500 feet along the paved path and then 500 feet in the sand to the house.
(b) Calculate the time T if you walk in the sand directly toward the ocean for 500 feet and then turn left and walk along the beach for 1500 feet to the house.
(c) Express the time T to get from the parking lot to the beachhouse as a function of the angle θ shown in the illustration.
(d) Calculate the time T if you walk directly from the parking lot to the house.
[**Hint**: $\tan \theta = 500/1500$]
(e) Calculate the time T if you walk 1000 feet along the paved path and then walk directly to the house.
(f) Graph $T = T(\theta)$. For what angle θ is T least? What is x for this angle? What is the minimum time?
(g) Explain why $\tan \theta = \dfrac{1}{3}$ gives the smallest angle θ that is possible.

68. **Carrying a Ladder around a Corner** Two hallways, one of width 3 feet, the other of width 4 feet, meet at a right angle. See the illustration.
(a) Express the length L of the line segment shown as a function of the angle